

# A HEURISTIC APPROACH FOR PROVIDER SELECTION AND TASK ALLOCATION MODEL IN TELECOMMUNICATION NETWORKS UNDER STOCHASTIC QOS GUARANTIES\*

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In this study, we model provider selection and task allocation problem as an expected cost minimization problem with stochastic chance constraints. Two important parameters of Quality of Service (QoS), delay and jitter are considered as random variables to capture stochastic nature of telecom network environment. As solution methodology, stochastic model is converted into its deterministic equivalent and then a novel heuristic algorithm is proposed to solve resulting nonlinear mixed integer programming model. Finally, performance of solution procedure is tested by several randomly generated scenarios.

## 1. Introduction

We consider an environment in which the firm can acquire network capacity with different service qualities at different prices in order to complete its daily tasks [1]. Day-to-day operations such as video conferencing, voice over IP and data applications are allocated between these acquired capacities by considering QoS requirement of each operation. For telecommunication networks QoS offered by providers are measured in terms of delay, jitter, lost rate and latency [2]. Specifications of these measures are described in Service Level Agreements (SLA) [3]. In this paper, two important parameters of QoS, delay and jitter are considered as random variables to capture stochastic nature of telecom network environment. The resulting optimization problem is nonlinear mixed integer problem with probabilistic constraints which is considered as NP-hard. Therefore, a heuristic algorithm capable of solving large problem instances is developed.

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The rest of the paper is organized as follows. Section 2 presents problem definition, notation and the proposed nonlinear mixed integer programming model with chance constraints [4, 5]. Heuristic algorithm is summarized in Section 3 to unravel the problem. Results of computational studies are also demonstrated in this section. Finally, Section 4 draws conclusions and describes future goals respectively.

## 2. Stochastic and Deterministic Mathematical Models

We will use parameters and decision variables given in Table 1 while presenting formulations and solution procedures throughout the text. Each formulation uses a subset of these variables and parameters.

Table 1. Problem notations and parameters

Parameters	
$I, J:$	The ordered index set of resources, and tasks respectively.
$A_T, A_S:$	The index set of tasks with fixed transmission time and fixed size respectively where $A_T \cap A_S = \emptyset$
$\delta_j:$	The maximum amount of delay tolerated by task $j$ .
$\rho_j:$	The maximum amount of jitter tolerated by task $j$
$D_i:$	Amount of delay in resource $i$ , which is a random variable and has a probability distribution $F^D_i(.)$
$R_i$	Amount of jitter in resource $i$ , which is a random variable and has a probability distribution $F^R_i(.)$
$\alpha_i:$	Transmission efficiency, calculated as one minus the packet loss rate of resource $i$ .
$\varphi_i:$	The maximum allowed overflow probability to assign a task into resource $i$ .
$p_j^\delta, p_j^\rho:$	Confidence level parameters for delay and jitter respectively for task $j$ .
$\beta_i, L_i:$	The bandwidth and duration of resource $i$ . $L_i = \min$ (length of contract, planning horizon)
$c_i:$	The total cost of resource $i$ for specific $\beta_i, L_i$ .
$c_j^o:$	The opportunity cost of missing the target transmission rate for task $j$ .
$c_j^\delta:$	The opportunity cost of not meeting minimum quality requirement for delay in task $j$ .
$c_j^\rho:$	The opportunity cost of not meeting minimum quality requirement for jitter in task $j$ .
$q_i:$	The unit overflow cost for going beyond the length of contract period for resource $i$
$-U_{r_j}, -L_{r_j}:$	Target and the minimum transmission rate of task $j$ at the receiving node, respectively, $j \in A_T$ .
$\Delta t_j:$	Estimated scheduled transmission time for time-fixed task $j \in A_T$ .

$\bar{x}_j$ :	The (fixed) length of task $j \in A_s$ in number of bits.
$\mu_i^D, \sigma_i^D$ :	Mean and standard deviation of probability distribution $F_i^D(.)$ .
$\mu_i^R, \sigma_i^R$ :	Mean and standard deviation of probability distribution $F_i^R(.)$ .
$\psi(a, b, c)$ :	Return value of integral $\int_a^\infty x \frac{1}{\sqrt{2\pi c^2}} e^{-\frac{(x-b)^2}{2c^2}} dx$
$\phi(a, b, c)$ :	Return value of integral $\int_a^\infty \frac{1}{\sqrt{2\pi c^2}} e^{-\frac{(x-b)^2}{2c^2}} dx$
<b>Decision variables</b>	
$v_i$ :	1, if resource $i$ is selected 0, otherwise
$r_j$ :	The transmission rate of task $j$ .
$y_{ij}$ :	1 if task $j$ is assigned to resource $i$ , 0 otherwise.
$y_{ijt}$ :	1 if task $j$ is active (transmitting) at time $t$ on resource $i$ , 0 otherwise
$t_j$ :	Start time of task $j$ .

## 2.1. Stochastic Problem (SP) Formulation

We model the capacity and the loss probability requirements explicitly, but formulate delay and jitter as random variables and added stochastic chance constraints associated with marginal probability distributions. We consider the all-you-can-send pricing scheme [1, 6] in which the firm pays a fixed price for a fixed bandwidth available for a fixed duration. We also assume that a real-time task incurs an opportunity cost if its transmission rate falls below a desired level.

$$\begin{aligned} \text{Minimize } z = & \sum_i c_i v_i + \sum_i \sum_{j \in A_T} (\bar{r}_j^U - r_j \alpha_i) c_j^\rho y_{ij} + \sum_i \sum_{j \in A_T \cup A_S} y_{ij} \mathbb{E}_{D_i} [c_j^\delta \text{Max}(D_i - \delta_j, 0)] \\ & + \sum_i \sum_{j \in A_T \cup A_S} y_{ij} \mathbb{E}_{R_i} [c_j^\rho \text{Max}(R_i - \rho_j^\sigma, 0)] + \sum_i \sum_{j \in A_T \cup A_S} y_{ij} \mathbb{E}_{D_i} \left[ q_i \text{Max} \left( t_j + \Delta t_j + \right. \right. \\ & \left. \left. D_i - L_i, 0 \right) \right] \end{aligned} \quad (1)$$

$$\text{Subject to } \sum_{j \in A_S} \bar{x}_j y_{ij} + \sum_{j \in A_T} \alpha_i \Delta t_j r_j y_{ij} \leq \alpha_i \beta_i L_i \quad \forall i \in I \quad (2)$$

$$\Pr \{ y_{ij} D_i \leq \delta_j \} \geq p_j^\delta \quad \forall j \in J, \forall i \in I, \quad 0 \leq p_j^\delta \leq 1 \quad (3a)$$

$$\Pr \{ y_{ij} R_i \leq \rho_j \} \geq p_j^\sigma \quad \forall j \in J, \forall i \in I, \quad 0 \leq p_j^\sigma \leq 1 \quad (3b)$$

$$\Pr \{ (t_j + \Delta t_j + D_i) y_{ij} \leq L_i \} \geq \phi_i \quad \forall j \in A_T, \forall i \in I \quad (4)$$

$$\sum_{j \in J} r_j y_{ijt} \leq \beta_i \quad \forall i \in I, t = 1..L_i \quad (5)$$

$$\sum_{i \in I} y_{ij} = 1, \quad \forall j \in J \quad (6)$$

$$\sum_{i \in I} \sum_{t \leq L_i} y_{ijt} = \Delta t_j, \quad \forall j \in A_T \quad (7)$$

$$\sum_{i \in I} \sum_{t \leq L_i} r_j y_{ijt} = \bar{x}_j, \quad \forall j \in A_S \quad (8)$$

$$y_{ij} \leq v_i \quad \forall i \in I, j \in J \quad (9)$$

$$y_{ijt} \leq y_{ij}, \quad \forall i \in I, j \in J, t \leq L_i \quad (10)$$

$$r_j \alpha_i \geq y_{ij} \bar{r}_j^L \quad \forall j \in A_T, \forall i \in I \quad (11)$$

$$r_j \alpha_i y_{ij} \leq \bar{r}_j^U \quad \forall j \in A_T, \forall i \in I \quad (12)$$

$$r_j, t_j \geq 0 \quad (13)$$

$$v_i, y_{ij}, y_{ijt} \in \{0, 1\} \quad (14)$$

Objective function(1) of problem SP considers the tradeoff between the costs of acquiring capacity, the cost of not meeting target transmission rates in real-time tasks, expected opportunity cost for not meeting minimum quality specifications for delay and jitter, and expected overflow cost due to not completing transmission any task assigned within contract period. We assume that available capacity is purchased at a fixed price for a specific bandwidth and duration (all-you-can-send pricing). Constraint set (2) guarantees that we can use only up to available capacity. Stochastic Constraint sets (3a) and (3b) ensure that the resources satisfy the minimum QoS requirements of the tasks that are assigned to it within determined confidence limits. Constraint (4) ensures with  $\varphi_i$  probability that all time-fixed tasks assigned to resource  $i$  are completed within the contracted duration of resource  $i$ . Note that a similar constraint is not needed for size-fixed tasks since constraint set (8) guarantees that a size-fixed task is only assigned if there is enough capacity and can be completed on time since we can arbitrarily vary the transmission rate (i.e. morph its shape). Constraint set (5) prevents using more bandwidth than available at any time (bandwidth dimension). Constraint (6) along with (14) ensures that a task is assigned to only one resource and all tasks are assigned. Constraint sets (7) guarantee that the tasks are actually allocated the required amount of time slices. Constraint set (9) guarantees that a network resource is selected only if at least one task is assigned to it. Constraint set (10) ensures that a task is assigned to a network resource only if it occupies a time slice on it. Constraint set (11) states that transmission rate for a time-fixed task  $j$  should be high enough to satisfy the

minimum transmission rate at the sink node. Constraint set (12) enforces the target transmission limitation for all tasks.

## 2.2. Deterministic Equivalent of Stochastic Problem (DESP)

Terms in expected form in Eq. (1) can be transferred into deterministic domain as following:

$$\mathbb{E}_{D_i} [Max(D_i - \delta_j, 0)] = \Psi(\delta_j, \sigma_i^D, \mu_i^D) - \delta_j \phi(\delta_j, \sigma_i^D, \mu_i^D) \quad \forall i \in I, \forall j \in J$$

$$\mathbb{E}_{R_i} [Max(R_i - \rho_j, 0)] = \Psi(\rho_j, \sigma_i^R, \mu_i^R) - \rho_j \phi(\rho_j, \sigma_i^R, \mu_i^R) \quad \forall i \in I, \forall j \in J$$

$$\mathbb{E}_{D_i} [Max(t_j + \Delta t_j + D_i - L_i, 0)] = \Psi(L_i - t_j - \Delta t_j, \sigma_i^D, \mu_i^D) - (t_j + \Delta t_j - L_i) \phi(L_i - t_j - \Delta t_j, \sigma_i^D, \mu_i^D) \quad \forall i \in I, \forall j \in J$$

All stochastic equations namely Eq. (3a), (3b) and (4) in SP can be converted in to linear constraints Eq.(15), (16) and (17), respectively:

$$y_{ij} \left( \frac{\delta_j - \mu_i^D}{\sigma_i^D} - Z_{p_j^\delta} \right) \geq 0, \quad \forall i \in I, \forall j \in J \quad (15)$$

$$y_{ij} \left( \frac{\rho_j - \mu_i^R}{\sigma_i^R} - Z_{p_j^\rho} \right) \geq 0, \quad \forall i \in I, \forall j \in J \quad (16)$$

$$y_{ij} \left( \frac{L_i - t_j - \Delta t_j - \mu_i^D}{\sigma_i^D} - Z_{\phi_i} \right) \geq 0, \quad \forall i \in I, \forall j \in J \quad (17)$$

## 3. Heuristic Solution and Computational Results

Pseudo-code of the heuristic algorithm (HDESP) for DESP is given in Figure 1 and initial results of algorithm are summarized in Table 2.

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Sort tasks by descending  $\rho$  and  $\delta$ , then Sort tasks by descending  $p^\delta, p^\rho$ 
  Sort above list by decreasing size of tasks
  Sort resources by increasing unit cost  $\bar{c} = c / (1 - \alpha) L \beta$ 
While all tasks not assigned
  If Eqs. 13-15 is satisfied, assign task to resource
  Else open new resource and make assignments
End while
Sort selected resources by decreasing unused capacity
  While enough unused capacity
    While new  $z \leq z$ 
      Swap tasks among selected resources
    End While
  Close unused resource
End While

```

Figure 1. Pseudo-code for HDESP



Table 2. Effect of Pricing Type on Cost Components

Pricing Type	Opportunity Cost	Quality Loss Cost	Capacity Leasing Cost	Total Cost
Random	94.34	35.96	449.11	579.41
Parallel cost curve	128.01	45.14	877.80	1050.95
Intersecting cost curve	129.11	47.51	1072.20	1248.82

#### 4. Conclusion and Future Study

We presented a novel formulation to solve the firm's network resource acquisition problem subject to stochastic QoS parameters, opportunity and quality loss cost. We also proposed a heuristic to handle QoS requirements and chance constraints. The proposed approach is able to solve moderate size problem sets in reasonable time limits which is not possible with standard commercial mathematical programming softwares. As a next step, quality of solutions generated by HDESP will be compared with calculated lower bounds.

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